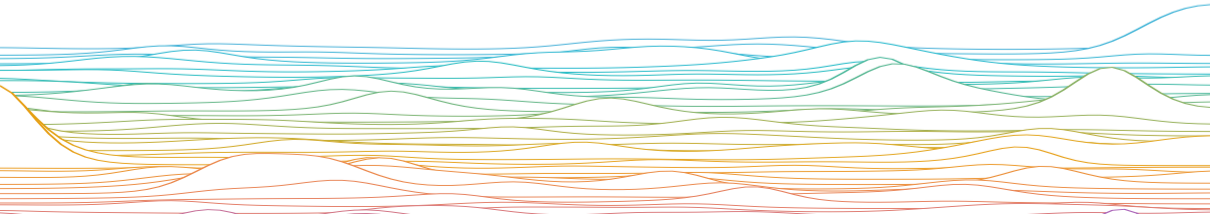


# Self-Healing Neural Codes

Changing to stay the same

M. Rule

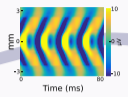
A job talk for lecturer in machine learning at the University of Edinburgh School of Informatics  
20th June, 2022



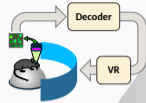
**Carnegie-Mellon University**  
**B.S. Computer Science**  
**Neural field theory**

**Brown University**  
**Ph.D. Neuroscience**  
**Statistical Modelling**

*Waves in cortex*

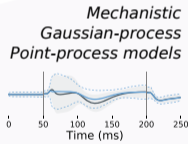


*Collective dynamics in motor cortex*



**Learning in closed-loop**

*Flicker-phosphene hallucinations*

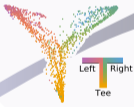
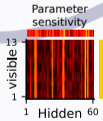


Probability of Grid Field

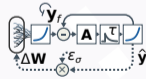


**Point-process methods**

*Statistics of and representations in Boltzmann machines*



*Drift, homeostasis, and learning*



**Biological Learning**

**University of Edinburgh (I)**  
**Statistical methods**  
**Past**

**Control Group**  
**University of Cambridge**  
**Homeostasis, Learning: Closed-loop**  
**Present**

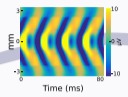
**University of Edinburgh (II)**  
**Future**

**Carnegie-Mellon University**  
**B.S. Computer Science**  
**Neural field theory**

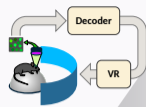
**Brown University**  
**Ph.D. Neuroscience**  
**Statistical Modelling**



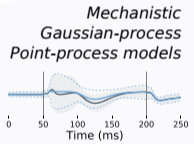
*Waves in cortex*



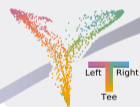
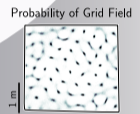
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**Learning in closed-loop**



**Point-process methods**

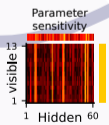


*Drift, homeostasis, and learning*



**Biological Learning**

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**Control Group**  
**University of Cambridge**  
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**Present**

**University of Edinburgh (I)**  
**Statistical methods**  
**Past**

**University of Edinburgh (II)**  
**Future**

*How do real neurons learn?*

*The brain is plastic.*  
*The brain remembers.*

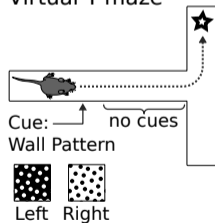
# How is the brain changing?

Driscoll & al. 2017 experiments:

- ▶ Image neural population activity over time in a fixed virtual-reality task



Virtual T-maze



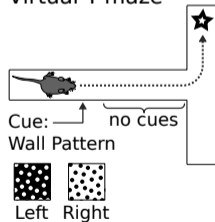
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Driscoll & al. 2017 experiments:

- ▶ Image neural population activity over time in a fixed virtual-reality task
- ▶ Posterior Parietal Cortex (PPC): Association area required for task



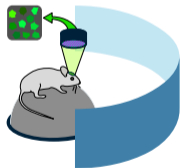
Virtual T-maze



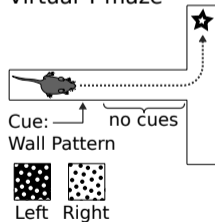
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- ▶ Long-term recording at steady state: task performed at expert level



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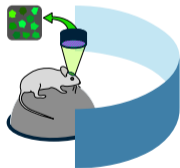




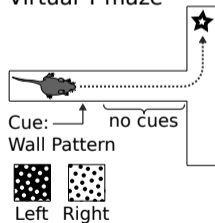
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- ▶ No change in performance or behavior



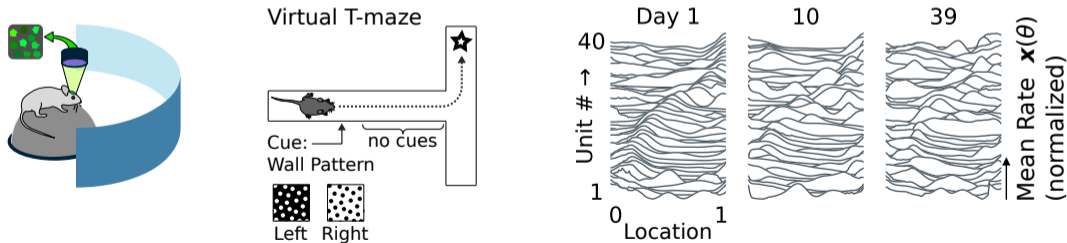
## Virtual T-maze



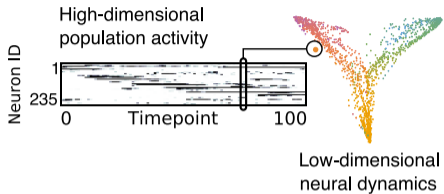
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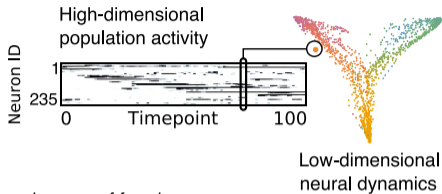
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- ▶ No change in performance or behavior
- ▶ Neural population code “drifts”

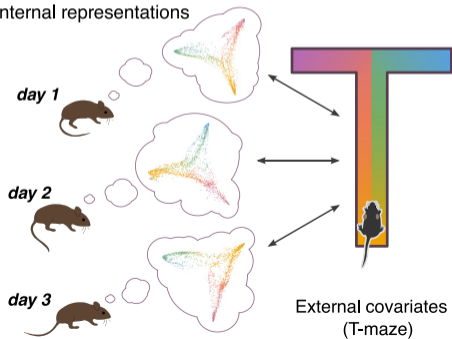


*... Neurons' roles change over time, even in representations supporting fixed, habitual tasks.*

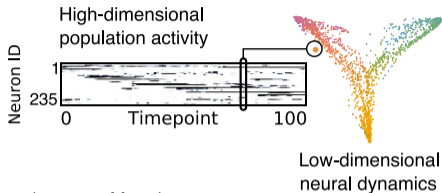




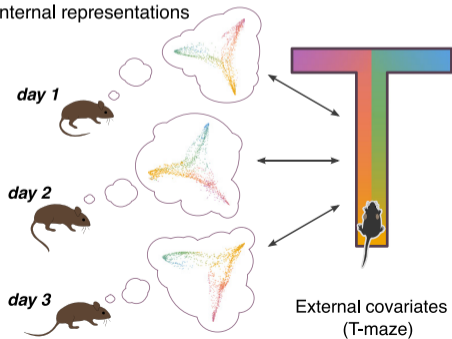
Many degrees of freedom  
in internal representations



Rule, M. E., O'Leary, T., and Harvey, C. D. (2019). Causes and consequences of representational drift. Current opinion in neurobiology, 58:141–147



Many degrees of freedom  
in internal representations

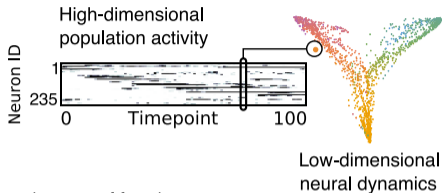


Ubiquitous, not uniform:

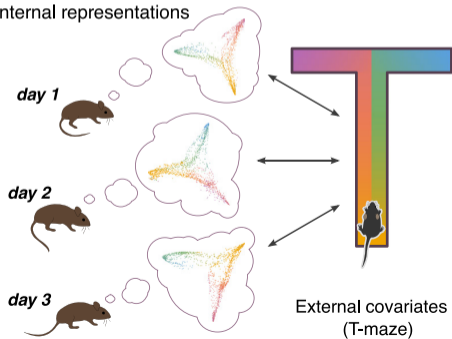
- ▶ **Hippocampus:**<sup>1,2</sup> Fast turnover; Episodic memory?
- ▶ **Olfactory:**<sup>3</sup> Experience-dependent: A code for novelty?
- ▶ **Visual**<sup>4</sup>, **Prefrontal:**<sup>5</sup>: More stable near periphery?

<sup>1</sup>Ziv & al. (2013); <sup>2</sup>Levy & al. (2021); <sup>3</sup>Schoonover, Fink & al. (2020);  
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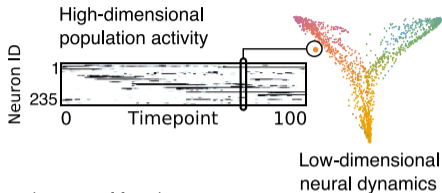
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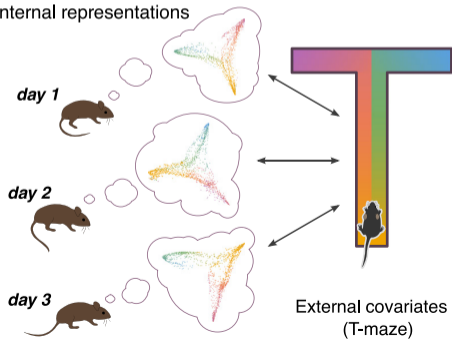
*What can drift tell us about learning?*

- ▶ Learning in over-parameterized networks?
- ▶ Strategies for rapid learning?
- ▶ Representations in large generative models?
- ▶ Avoiding catastrophic forgetting in continual learning?

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*How does the brain achieve stable behavior despite internal change?*

*Could other neurons read a code like this?*



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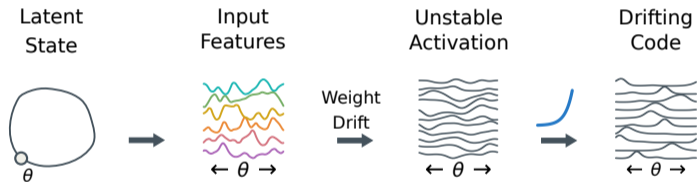
***Proposition:***

- ▶ Homeostasis stabilizes single neuron properties **and learned associations**
- ▶ Hebbian plasticity maintains learned representation **by re-enforcing existing correlation** structure

Supported by  
L E V E R H U L M E  
T R U S T \_\_\_\_\_  
early-career fellowship

# Unstable code, stable latent structure: A problem for long-term stability?

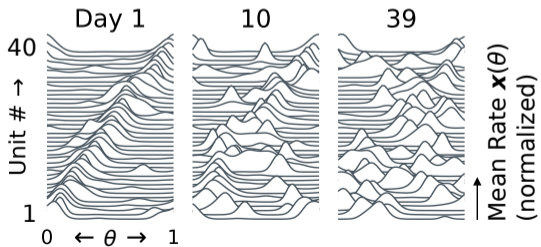
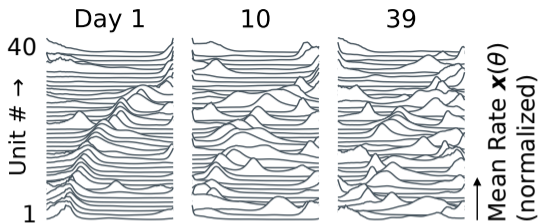
*Model drift:* Random features + drifting weights + homeostatic normalization



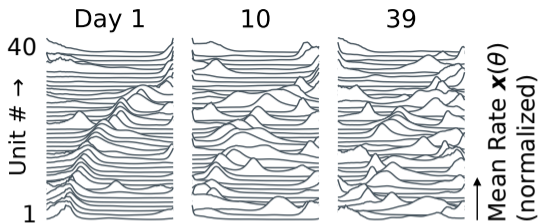
*Sensitivity & threshold homeostasis  
preserve localized tuning<sup>†</sup>*



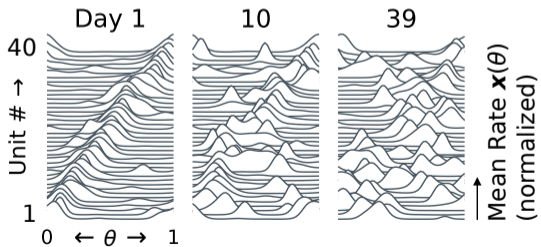
<sup>†</sup> **Homeostasis:** Cannon & Miller (2017, 19),

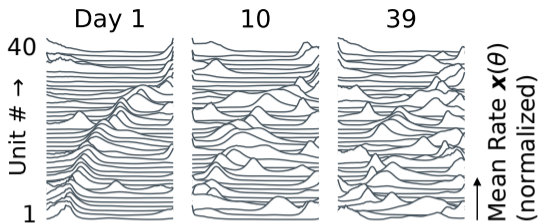


Simulated drift: which is data, which is model?

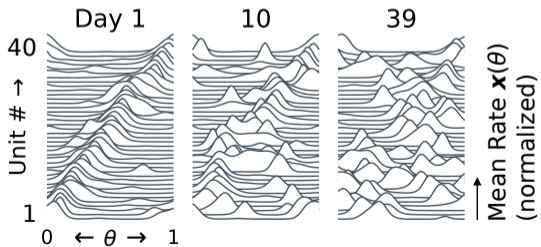


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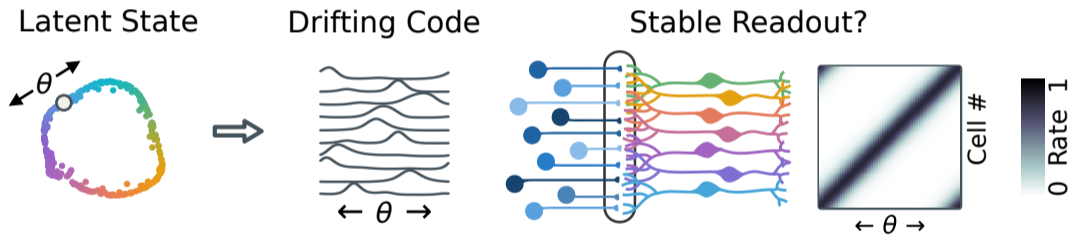
This one is data



This one is model

Simulated drift: which is data, which is model?

# Is a stable readout possible?



A readout population learns to decode location from an encoding population that drifts.  
Can it preserve this readout as the encoding changes?

# Yes, with some ongoing learning



T. O'Leary



A. Loback



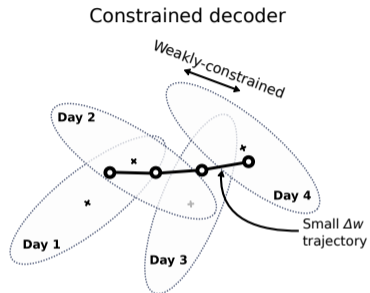
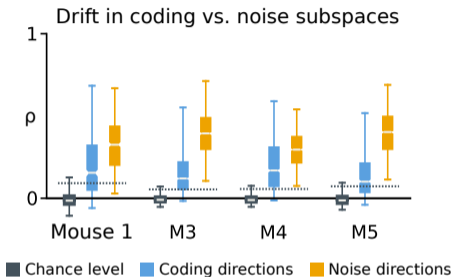
D. Raman



L. Driscoll



C. D. Harvey



Rule, M. E., Loback, A. R., Raman, D. V., Driscoll, L. N., Harvey, C. D., and O'Leary, T. (2020). Stable task information from an unstable neural population. *Elife*, 9:e51121

# Yes, with some ongoing learning



T. O'Leary



A. Loback



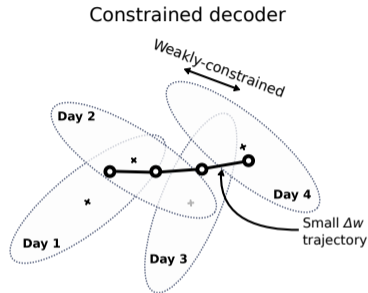
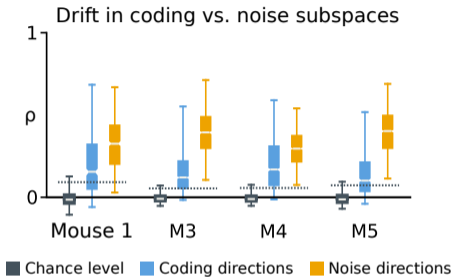
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- Most drift looks like trial-to-trial variability on slow time-scales



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T. O'Leary



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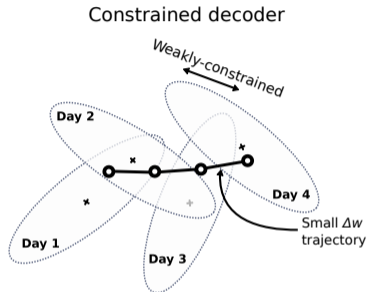
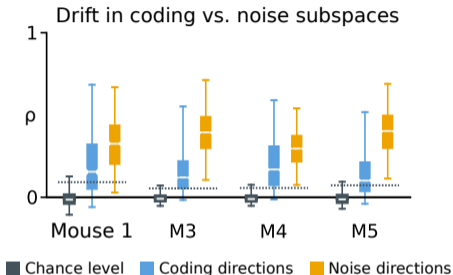
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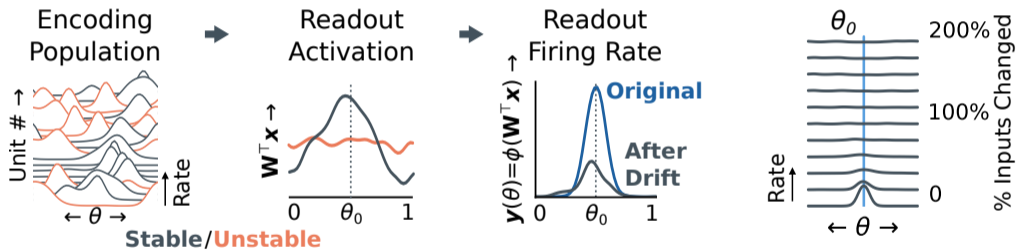


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- ▶ Most drift looks like trial-to-trial variability on slow time-scales
- ▶ Change in the underlying representation is gradual
  - Only small weight changes needed to stabilize decoding

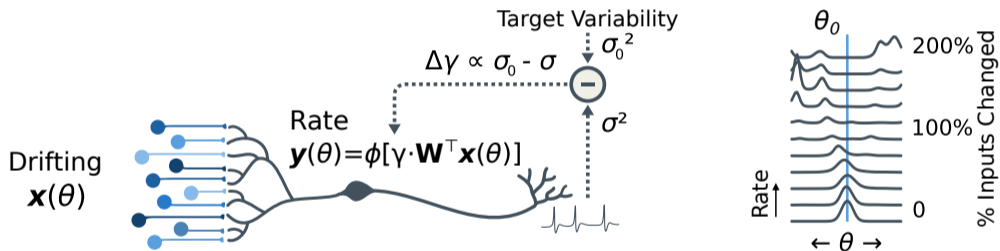
*Can we track a drifting code for continuous variables  
without an external reference?*

## Fixed weights: Drift attenuates excitatory drive



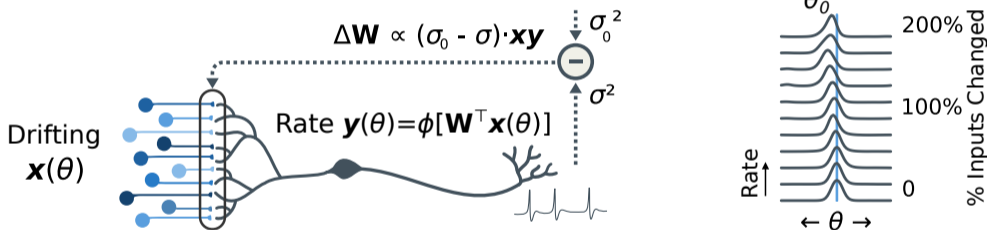
Readout neurons require a specific conjunction of inputs to fire.  
Random drift destroys this excitatory drive, but doesn't change tuning.

## Sensitivity homeostasis: Tuning is robust to modest change



Homeostatically scaling up weights stabilizes activity against small amounts of drift, since the input is somewhat redundant.

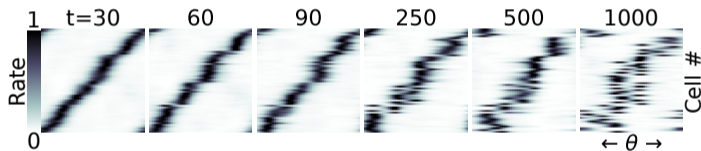
## Hebbian homeostasis: Use the neuron's own output as a training signal



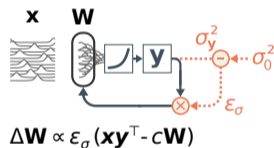
Using the neuron's own output to adjust weights stabilizes localized, bump-like tuning. This uses unsupervised Hebbian learning to homeostatically restore activity as drift destroys excitatory drive.

# Use recurrent predictions as a learning signal

## Hebbian Homeostasis



## Schematic

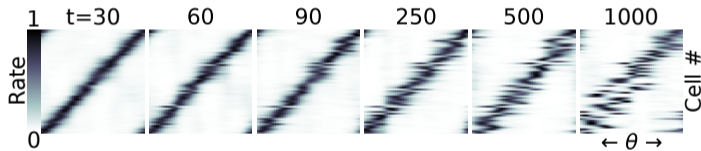


**Hebbian Homeostasis** provides stability, but tuning decays toward high-variance components<sup>†</sup>

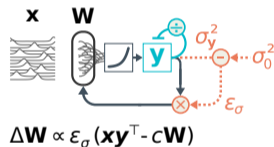
<sup>†</sup>c.f. Oja (1982)

# Use recurrent predictions as a learning signal

## Population Competition



## Schematic



**Hebbian Homeostasis** provides stability, but tuning decays toward high-variance components<sup>†</sup>

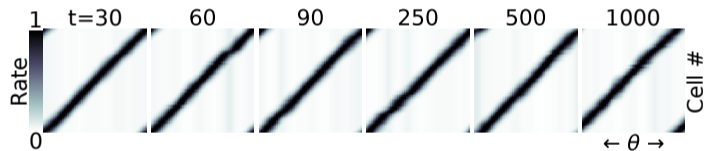
**Competition** between readout cells stabilizes further<sup>‡</sup>

<sup>†</sup>c.f. Oja (1982)

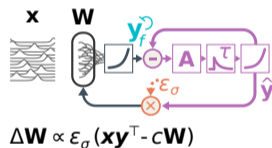
<sup>‡</sup>e.g. Földiák & Fdirl, Barlow & Földiák (1989); Pehlevan & al. (2015, 17, 19, 20)

# Use recurrent predictions as a learning signal

## Recurrent Dynamics



## Schematic



**Hebbian Homeostasis** provides stability, but tuning decays toward high-variance components<sup>†</sup>

**Competition** between readout cells stabilizes further<sup>‡</sup>

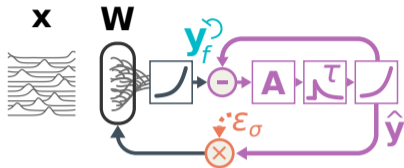
**Recurrent predictions** improve error-correction → long-term stability

<sup>†</sup>c.f. Oja (1982)

<sup>‡</sup>e.g. Földiák & Fdirl, Barlow & Földiák (1989); Pehlevan & al. (2015, 17, 19, 20)



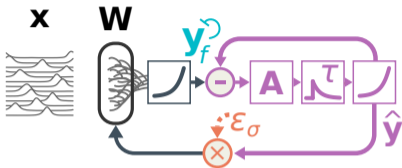
# Repair readout using a predictive model stored in recurrent weights



Generative model of spikes  $\mathbf{y} = \{y_1, \dots, y_n\}$

- ▶ Latent variables  $\mathbf{z} = \{z_1, \dots, z_n\} \sim P_z$
- ▶ Observation model  $P_{\mathbf{y}|\mathbf{z}}$
- ▶  $P_{\mathbf{y}} = \int_{d\mathbf{z}} P_{\mathbf{y}|\mathbf{z}} P_z$

## Repair readout using a predictive model stored in recurrent weights

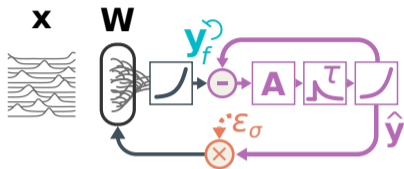


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- ▶ Latent variables  $\mathbf{z} = \{z_1, \dots, z_n\} \sim P_z$
- ▶ Observation model  $P_{\mathbf{y}|\mathbf{z}}$
- ▶  $P_{\mathbf{y}} = \int_{d\mathbf{z}} P_{\mathbf{y}|\mathbf{z}} P_z$

Learn a prior on likely states in latent variables  $\mathbf{z}$ , and how to map  $\mathbf{z} \leftrightarrow \mathbf{y}$ . Save this prior, and use it to repair readouts in the face of drift.

# Repair readout using a predictive model stored in recurrent weights



Specifically, a Log-Gaussian-Poisson model:

$$P_{\mathbf{z}} = \mathcal{N}[0, \Sigma] \propto \exp\left(-\frac{1}{2}\mathbf{z}^\top \Sigma^{-1} \mathbf{z}\right)$$

$$P_{y_i|z_i} = \text{Poisson}[\lambda_i = e^{z_i}] \propto \lambda_i^{y_i} e^{-\lambda_i}$$

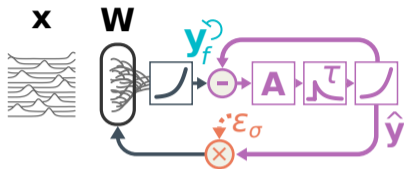
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Generative model of spikes  $\mathbf{y} = \{y_1, \dots, y_n\}$

- ▶ Latent variables  $\mathbf{z} = \{z_1, \dots, z_n\} \sim P_{\mathbf{z}}$
- ▶ Observation model  $P_{\mathbf{y}|\mathbf{z}}$
- ▶  $P_{\mathbf{y}} = \int_{d\mathbf{z}} P_{\mathbf{y}|\mathbf{z}} P_{\mathbf{z}}$

Learn a prior on likely states in latent variables  $\mathbf{z}$ , and how to map  $\mathbf{z} \leftrightarrow \mathbf{y}$ . Save this prior, and use it to repair readouts in the face of drift.

# Repair readout using a predictive model stored in recurrent weights



Generative model of spikes  $\mathbf{y} = \{y_1, \dots, y_n\}$

- ▶ Latent variables  $\mathbf{z} = \{z_1, \dots, z_n\} \sim P_z$
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Specifically, a Log-Gaussian-Poisson model:

$$P_z = \mathcal{N}[0, \Sigma] \propto \exp\left(-\frac{1}{2}\mathbf{z}^T \Sigma^{-1} \mathbf{z}\right)$$

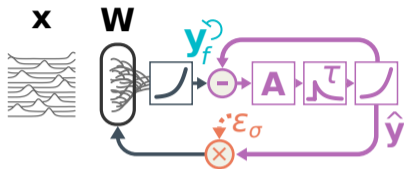
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Error correct: get  $\mathbf{z}$  most likely to explain  $\mathbf{y}$

$$\dot{\mathbf{z}} \propto \nabla_{\mathbf{z}} \ln P_{z|\mathbf{y}} = \mathbf{y} - \boldsymbol{\lambda}(\mathbf{z}) - \Sigma^{-1} \mathbf{z}$$

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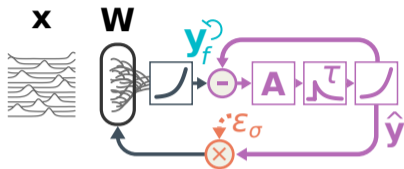
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Equivalent:  $\dot{\mathbf{z}} \propto -\mathbf{z} + \Sigma[\mathbf{y} - \boldsymbol{\lambda}(\mathbf{z})]$

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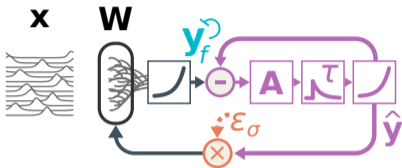
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Predictions  $\boldsymbol{\lambda}(\mathbf{z})$  cancel inputs  $\mathbf{y}$  via inhibition, through recurrent weights  $\Sigma$ : A rate network, but not quite physiological<sup>1</sup>.

<sup>1</sup>compare to Masset & al. (2022), which shares some assumptions;

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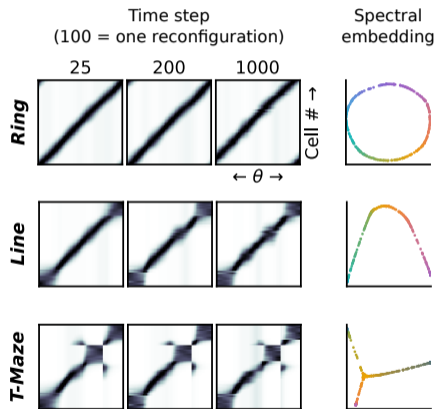
These questions remain open: Future work.

<sup>1</sup>compare to Masset & al. (2022), which shares some assumptions;

# Drift in low-dimensional representations is benign

*Homeostasis and internal predictions confer long-term stability to drift*

Rule, M. E. and O'Leary, T. (2022). Self-healing codes: How stable neural populations can track continually reconfiguring neural representations. Proceedings of the National Academy of Sciences, 119(7):e2106692119





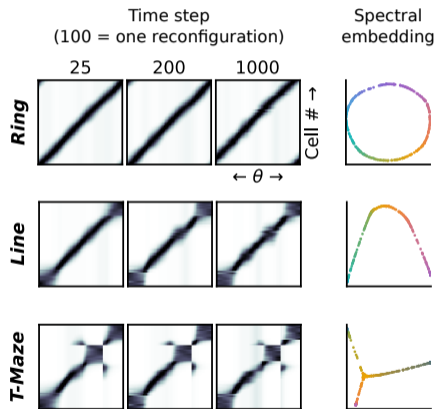
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**Drift:** Unstable code, stable low-D structure

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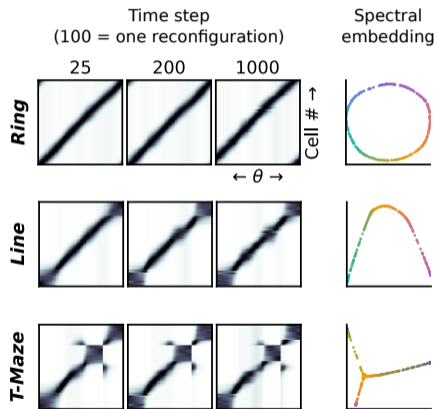
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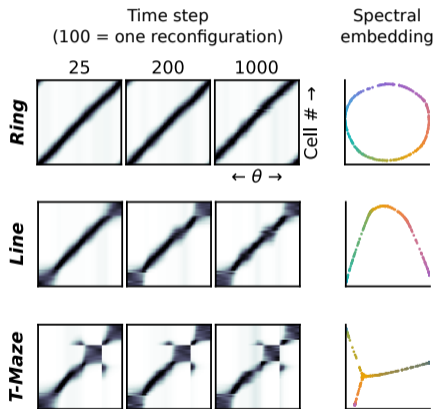
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*Recurrent population interactions*

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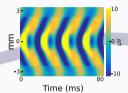


**Carnegie-Mellon University**  
**B.S. Computer Science**  
**Neural field theory**

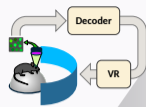
**Brown University**  
**Ph.D. Neuroscience**  
**Statistical Modelling**



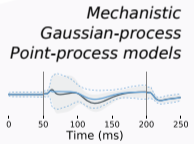
*Waves in cortex*



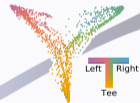
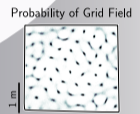
*Collective dynamics in motor cortex*



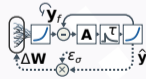
**Learning in closed-loop**



**Point-process methods**

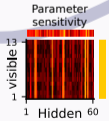


*Drift, homeostasis, and learning*



**Biological Learning**

*Statistics of and representations in Boltzmann machines*



**Control Group**  
**University of Cambridge**  
**Homeostasis, Learning: Closed-loop**  
**Present**

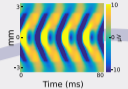
**University of Edinburgh (I)**  
**Statistical methods**  
**Past**

**University of Edinburgh (II)**  
**Future**

**Carnegie-Mellon University**  
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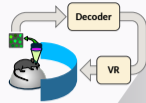
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**Learning in closed-loop**



*Flicker-phosphene hallucinations*

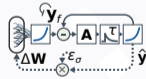


**Point-process methods**

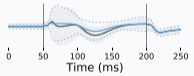
Probability of Grid Field



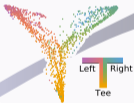
**Biological Learning**



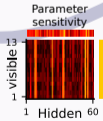
*Mechanistic Gaussian-process Point-process models*



*Drift, homeostasis, and learning*



*Statistics of and representations in Boltzmann machines*



**Control Group**  
**University of Cambridge**  
**Homeostasis, Learning: Closed-loop**  
**Present**

**University of Edinburgh (II)**  
**Future**

**University of Edinburgh (I)**  
**Statistical methods**  
**Past**



B. Ermentrout



W. Truccolo

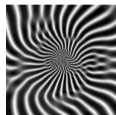
## Undergraduate (*Bard Ermentrout*): Neural-field theory

([Rule et al., 2011](#)) What do neural networks see?

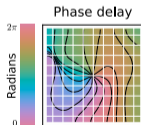
- ▶ 5–40 Hz flickering light: “phosphenes” hallucinations<sup>1</sup>
- ▶ To detect edges, visual cortex inhibits nearby inputs.
- ▶ Inverting input when inhibition arrives excites edges.
- ▶ Rate network with recurrent convolutions

([Heitmann, Rule et al. 2017](#)) Optogenetic stimulation switches cortex from travelling waves to oscillations.

<sup>1</sup>Purkinje 1819



**Neural-field theory simplifies collective activity for mathematical tractability.**



**Point-process models infer why neurons spike**

## Ph.D. (*Wilson Truccolo*): Point-process models

Collective Dynamics in Primate Motor Cortex

- ▶ ([Rule et al., 2017; 2018](#)) Before moving, rhythmic spiking encodes movement plans; Phase (re)alignments cause waves across cortex.
- ▶ ([Rule et al., 2015](#)) When moving, local electric fields relate to past/future movement; Past spiking predicts future variability.

*Can we combine these? Simplify point-process models, infer neural fields from spikes?*



*Autoregressive models predict the future from the past.*

Coarse graining (“zoom out”):

- ▶ Reduce the dimension of a large model
- ▶ Linear models with Gaussian noise are simpler to analyze

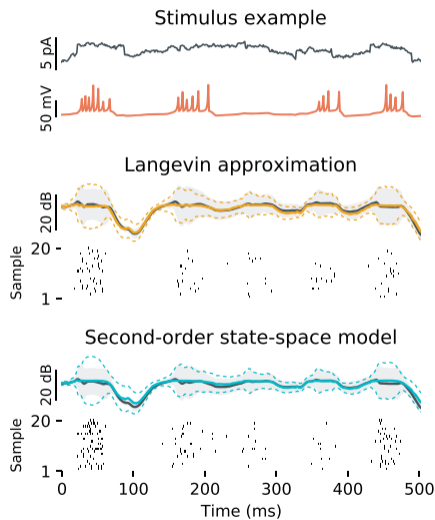
The probability of a neuron spiking can be predicted as a linear function of past variables, followed by a pointwise nonlinearity.

- ▶ Not Gaussian, but a Gaussian approximation is ok because predictions average over many spikes.

Process history  $\sim$  Gaussian process; Propagate in time using

- ▶ Moment-matching (“moment closure”), or
- ▶ Locally-quadratic approximation (improved stability)

Rule, M. and Sanguinetti, G. (2018). [Autoregressive point processes as latent state-space models: A moment-closure approach to fluctuations and autocorrelations](#). *Neural computation*, 30(10):2757–2780



# Neural field models for latent state inference



G. Sanguinetti D. Schnoerr M. H. Hennig E. Sernagor G. Hilgen

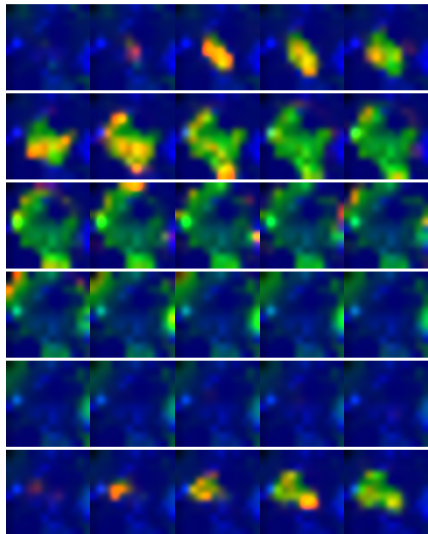
Neural field models are simple descriptions of spatiotemporal neural population dynamics.

- ▶ Partial Differential Equations (PDEs) over space and time.
- ▶ # neurons often  $\rightarrow \infty$  so that noise averages away.

Model noise in finite populations  $\rightarrow$  *stochastic* PDE.

- ▶ Use as a latent-variable model of waves in the retina.
- ▶ 3 states: Quiescent, Active, Refractory  
(compare to Susceptible, Infected, Recovered)
- ▶ Approximate latent distribution as a Gaussian process
- ▶ Estimate from spiking observations via Bayesian filtering

Rule, M. E., Schnoerr, D., Hennig, M. H., and Sanguinetti, G. (2019). [Neural field models for latent state inference: Application to large-scale neuronal recordings](#). *PLoS computational biology*, 15(11):e1007442



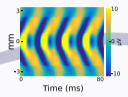


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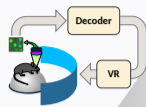
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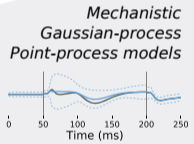
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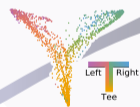


**Learning in closed-loop**

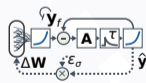


**Point-process methods**

Probability of Grid Field

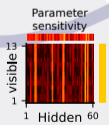


*Drift, homeostasis, and learning*



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# Are binary latent-variable models like the early sensory system?



*Restricted Boltzmann Machines (RBMs) are latent-variable models where the observed/latent variables are Bernoulli-distributed conditioned on each other.*

Martino Sorbaro was training RBMs to model population spiking activity, kept finding models close to “critical”.

Rare stimuli suppressed variability

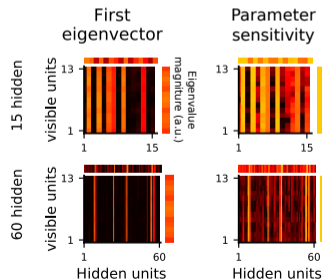
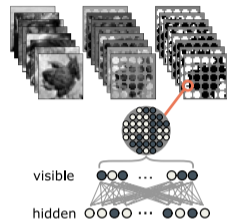
- ▶ Noise limits bandwidth; Rare stimuli need more bits to encode
- ▶ Incoming stimuli reduce neural variability<sup>1</sup>
- ▶ Suppression of firing can be informative<sup>2</sup>

$\infty$ -large models:  $1/f$  statistics and rank  $\approx 1$  Fisher information matrix<sup>3</sup>

- ▶ Implicit prior on model’s statistics
- ▶ Finite models also approximate this, given sufficient capacity.
- ▶  $\Rightarrow$  Easy to measure a weight’s importance from local activity

Rule, M. E., Sorbaro, M., and Hennig, M. H. (2020). *Optimal encoding in stochastic latent-variable models*. *Entropy*, 22(7):714

<sup>1</sup>Churchland & al. (2010), <sup>2</sup>Echeveste & al. (2020), <sup>3</sup>Schneidman (2011), <sup>3</sup>Schwab (2014)



# Can we understand developmental pruning via information theory?



*The brain removes neurons during maturation. How does it choose which ones?*

Fisher Information Matrix (FIM): amount a model changes when we change its parameters  $\approx$  Importance

- ▶ Only the diagonal can be detected locally. But, if  $FIM \approx \text{rank-1}$ , the diagonal is ok.

In Boltzmann machines, weight FI depends on pre/post activity

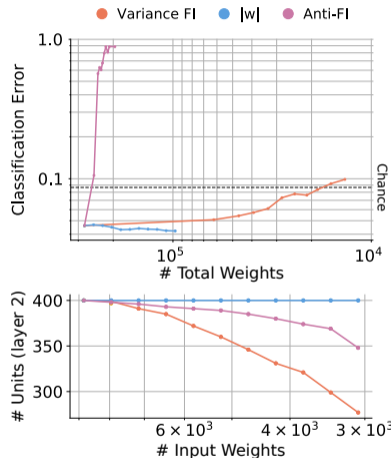
- ▶ More pre-post correlations  $\Rightarrow$  more important
- ▶ Prune small weights: **reduced** error, didn't shrink network
- ▶ FI pruning: good performance, shrinks network

## Must re-train post-pruning<sup>1</sup>

- ▶ Use homeostasis for “internal” transfer learning during pruning?

Scholl, C., Rule, M. E., and Hennig, M. H. (2021). *The information theory of developmental pruning: Optimizing global network architectures using local synaptic rules*. *PLoS computational biology*, 17(10):e1009458

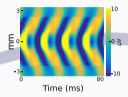
<sup>1</sup>Not the best approach for industrial ML: Crowley & al. (2018, 2019)



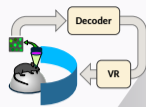
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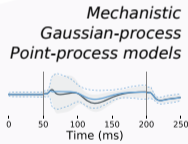


*Collective dynamics in motor cortex*



**Learning in closed-loop**

*Flicker-phosphene hallucinations*



*Mechanistic Gaussian-process Point-process models*



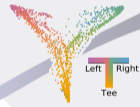
**Retinal waves**

**Point-process methods**

Probability of Grid Field



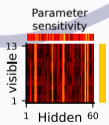
*Drift, homeostasis, and learning*



**Biological Learning**



*Statistics of and representations in Boltzmann machines*



**Control Group**  
**University of Cambridge**  
**Homeostasis, Learning: Closed-loop**  
**Present**

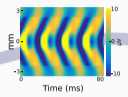
**University of Edinburgh (II)**  
**Future**

**University of Edinburgh (I)**  
**Statistical methods**  
**Past**

**Carnegie-Mellon University**  
**B.S. Computer Science**  
**Neural field theory**

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**Ph.D. Neuroscience**  
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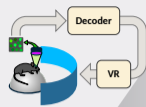
*Waves in cortex*



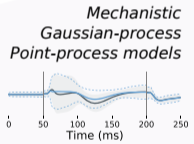
*Flicker-phosphene hallucinations*



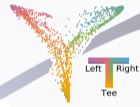
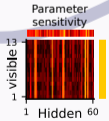
*Collective dynamics in motor cortex*



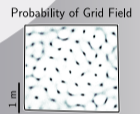
**Learning in closed-loop**



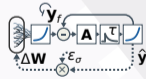
*Statistics of and representations in Boltzmann machines*



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**Point-process methods**



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J. Krupic



P. C. Vayalambone



T. O'Leary

“Grid cells” in hippocampal formation fire as a periodic (hexagonal) function of animal’s location “ $x$ ”.

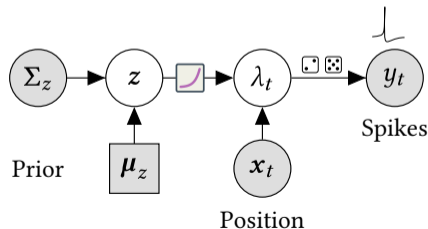
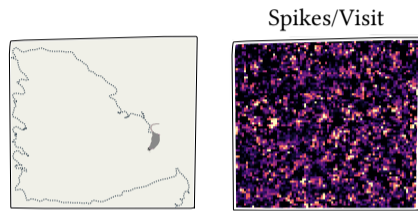
**Can we model distribution of firing-rate maps  $P_{\text{spike}}(\mathbf{x})$  from limited data?**

- ▶ (Latent) log-rate function  $z(\mathbf{x})$  with Gaussian-process prior  
 $P_{z(\mathbf{x})} = \mathcal{GP}(\boldsymbol{\mu}_0, \Sigma)$
- ▶ Point process observations:  
 $P_{\mathbf{y}(\mathbf{x})|z(\mathbf{x})} = \text{Poisson}[\boldsymbol{\lambda}(\mathbf{x}) = e^{z(\mathbf{x})} dx]$

Covariance kernel for periodic grids

- ▶ Bessel function; 0<sup>th</sup> order, 1<sup>st</sup> kind
- ▶ Multiply by a local window

M. Rule, P. Chaudhuri-Vayalambone, J. Krupic, T. O’Leary (2023). Variational log-Gaussian point-process modelling: Application to hippocampal grid cells. In Preparation.



# Gaussian processes for grid cells: Make it faster



J. Krupic



P. C. Vayalambone



T. O'Leary

Pandemic lockdown: run everything on an old laptop,  
 $2^{16}$  dimensions, large datasets, how?

Variational Bayes:

- ▶  $P_{z(x)|y(x)} \approx Q_{z(x)} = \mathcal{GP}(\mu, \Sigma)$
- ▶  $\Sigma^{-1}$  = prior + diagonal update

Fit iteratively

- ▶  $\hat{\mu} \leftarrow \underset{\mu}{\operatorname{argmax}} \langle \log\text{-prior} + \log\text{-likelihood} \rangle_Q^*$
- ▶  $\Sigma \leftarrow \Sigma_0 + \operatorname{diag}[\langle \lambda(x) \rangle]$

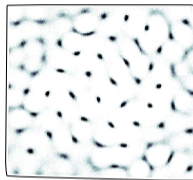
Evaluate on grid

- ▶ Bin data with interpolation to 'pseudopoints'
- ▶  $\Sigma_0$  becomes a convolution

Low-rank spatial frequency subspace

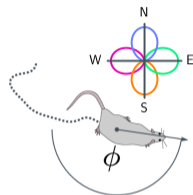
- ▶ Discard frequencies  $\approx 0$  in the prior
- ▶ (Fast) Hartley transform gives real-valued components compatible w. Krylov subspace solvers

Probability of Grid Field

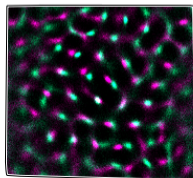


Heading-Dependent Weights

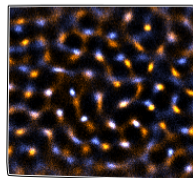
$$w = \cos(\phi - \phi_0)^2$$



Eastward (green) vs  
Westward (red)



Northward (blue) vs  
Southward (orange)





E. Sorrell



T. O'Leary



F. Forni



C. D. Harvey



D. Wilson

## Closed-loop control in an adaptive system

- ▶ How does the brain adapt motor control to new circumstances?

## Brain-Machine interface

- ▶ Force parietal cortex to act as a motor output
- ▶ **We** control the motor response to neuronal firing

## Decode directly from images

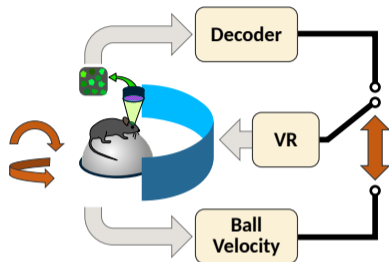
- ▶ Calibrate during behavior using stochastic gradient descent
- ▶ Test in closed loop

## It works

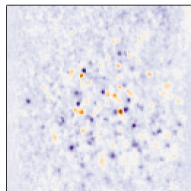
- ▶ No substantial tuning drift over five days
- ▶ Mean-rates of neurons become sensitive to BMI/control context

Sorrell, E., Rule, M. E., and O'Leary, T. (2021). Brain-machine interfaces: Closed-loop control in an adaptive system. Annual Review of Control, Robotics, and Autonomous Systems, 4

Sorrell, E, Wilson, D, Rule, M, Yang, H, Forni, F, Harvey, C, O'Leary, T. (2022) A Calcium Imaging Based Brain-Machine Interface for Virtual Navigation. In preparation.



Normalized weights  
(forward velocity)

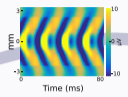




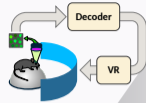
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*Waves in cortex*

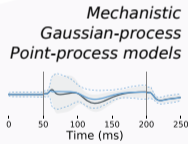


*Collective dynamics in motor cortex*



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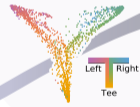
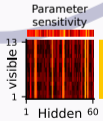
**Retinal waves**

Probability of Grid Field

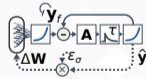


**Point-process methods**

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