A Model for the Origin and Properties of Flicker-Induced Geometric Phosphenes

SIAM Life Sciences

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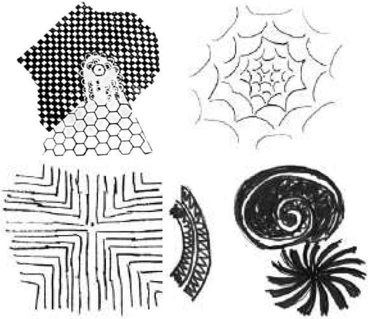
work performed under G. Bard Ermentrout,
in collaboration with Matthew Stoffregen

August 8, 2012

Motivations

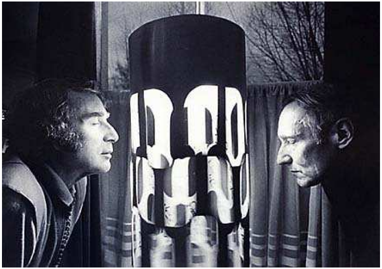
- Described by Jan Purkinje in 1819



Purkinje's Illustrations

Motivations

- Described by Jan Purkinje in 1819
- Artistic and broader interest
    - Flicker hallucinations



William S. Burroughs & Biron Gysin with the "Dream Machine"

Motivations

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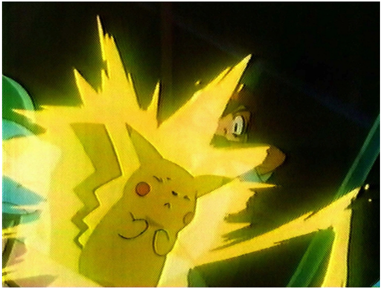
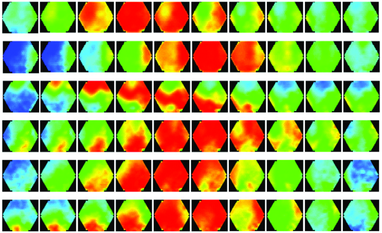
  - Flicker hallucinations
- Applications
  - Photosensitive epilepsy, migraines, vertigo



Pokemon television series

Motivations

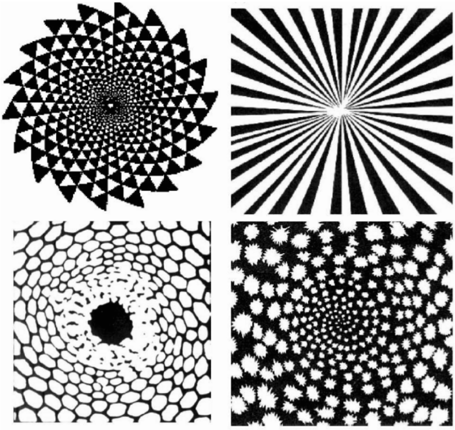
- Described by Jan Purkinje in 1819
- Artistic and broader interest
  - Flicker hallucinations
- Applications
  - Photosensitive epilepsy, migraines, vertigo
- Recurrent networks
  - Temporal encoding
  - Oscillations
  - Spatio-temporal coupling



Compression and Reflection of Visually Evoked Cortical Waves, Xu et al. 2007

What are geometric phosphene hallucinations?

- Form constants (Kluver, 1960)
  - **reproducible** across subjects



Form constants, Bressloff et al. 2001

What are geometric phosphene hallucinations?

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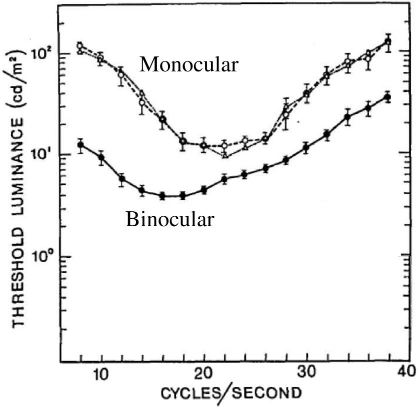
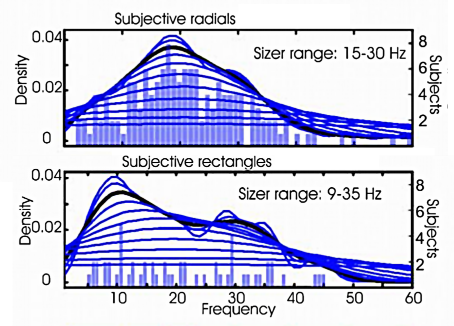
  - **reproducible** across subjects
- 10-40 Hz (Remole, 1971)



Remole, 1973

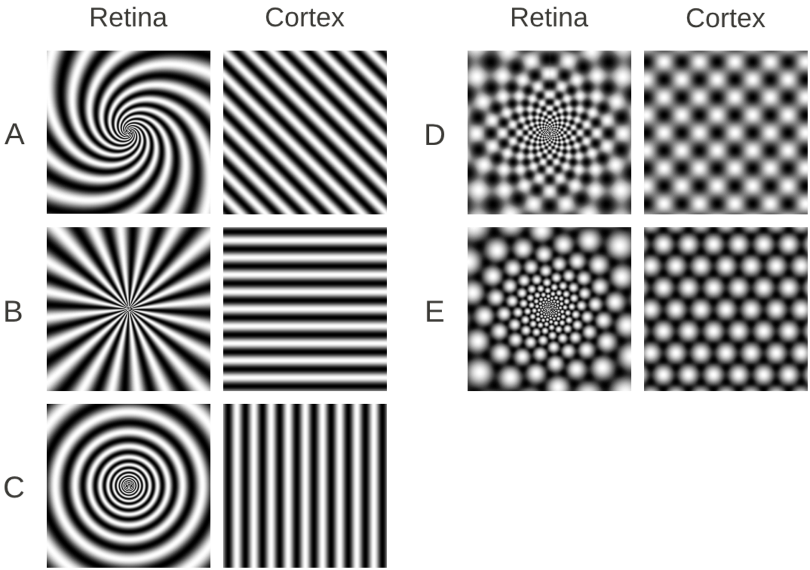
What are geometric phosphene hallucinations?

- Form constants (Kluver, 1960)
  - **reproducible** across subjects
- 10-40 Hz (Remole, 1971)
- Becker, Elliot (2006):
  - 10 Hz : honeycombs, rectangles, zigzags
  - 20-30 Hz: spirals, targets, lines, waves



Flicker-induced color & form: Interdependencies & relation to stimulation frequency & phase, Becker & Elliott, 2006

Form Constants : Waves in Cortex



Geometric visual hallucinations : instabilities in V1

- Ermentrout, Cowan (1979) : inhibition, excitation **instability**
  - Migraine
  - Sensory deprivation
  - Hallucinogens
- Instability when driven with 'unnatural' stimuli
  - Geometric phosphenes from **uniform flickering light**
- Knoll (1963) : flicker phosphenes relate to resonance?
- Herrmann (2001): 10,20,40 Hz **resonance** in occipital EEG to flickering light

Open problems addressed in this talk

- Can existing models of visual hallucination explain flicker-phosphenes?

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- Can existing models of visual hallucination explain flicker-phosphenes?
- How can spatially **uniform** stimuli lead to hallucinated **patterns**?
- Why do some stimuli induce hallucinations more readily than others?
- How do different **temporal** stimuli induce different **spatial** patterns?

Wilson-Cowan equations: excitatory, inhibitory populations

$$\tau_e \dot{U}_e = -U_e + f(a_{ee}U_e - a_{ie}U_i - \theta_e + g_e S(t))$$
$$\tau_i \dot{U}_i = -U_i + f(a_{ei}U_e - a_{ii}U_i - \theta_i + g_i S(t))$$

$U_{e,i}$: Population activation

$\tau_{e,i}$: Time constants

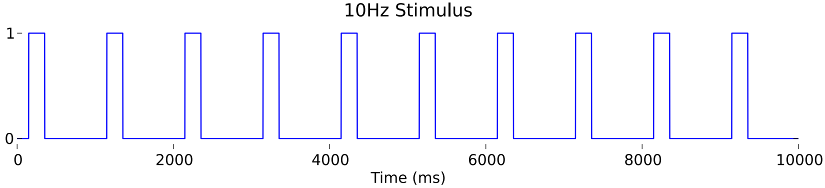
$a_{ee,ei,ie,ii}$: Population interaction

$\theta_{e,i}$: Bias

$g_{e,i}$: Stimulus coupling

Firing rate nonlinearity: $f(x) = \frac{1}{1+e^{-x}}$

Periodic stimulus: $S(t) = H(sin(2\pi t/T) - 0.8)$



Model: spatially extended Wilson-Cowan equations

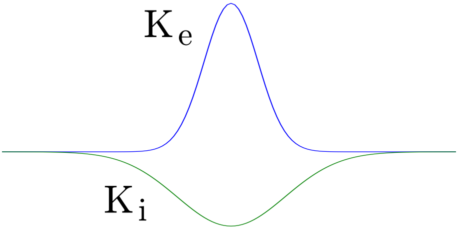
$$\tau_e \dot{U}_e(x,t) = -U_e(x,t) + f(a_{ee} K_e \star U_e(x,t) - a_{ie} K_i(x) \star U_i(x,t) - \theta_e + g_e S(t)$$

$$\tau_i \dot{U}_i(x,t) = -U_i(x,t) + f(a_{ei} K_e \star U_e(x,t) - a_{ii} K_i(x) \star U_i(x,t) - \theta_i + g_i S(t))$$

(based on Ermentrout and Cowan 1979)

Lateral interaction kernel

$$K_{e,i} \propto e^{-x^2/\sigma_{e,i}^2}$$



Notation

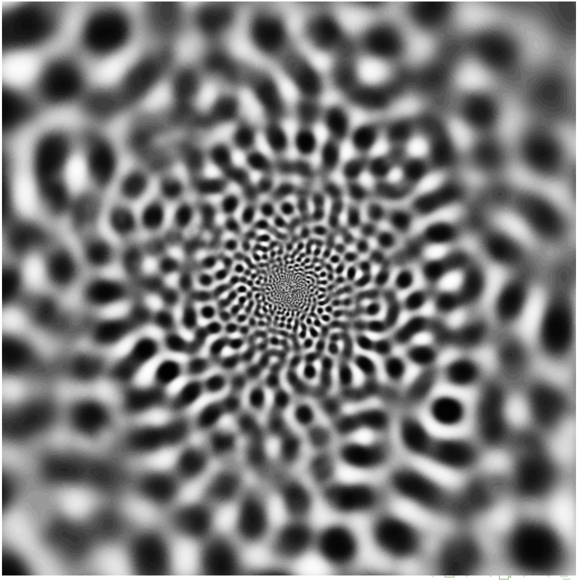
For succinctness, we sometimes denote this system as

$$\dot{U} = -DU + F(KU + GS(t))$$

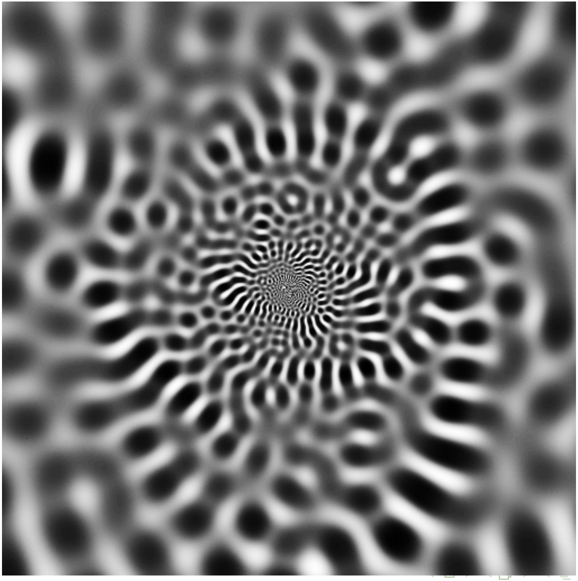
- D : matrix of time constants
- F : nonlinearity applied in each dimension
  - offsets θ subsumed in to the nonlinearity,
- K : matrix of interactions,
  - including lateral interactions and e–i interactions
- $GS(t)$: stimulus drive to each dimension.

Simulation

Evolution of 2D patterns (subjective coordinates)



Evolution of 2D patterns (subjective coordinates)



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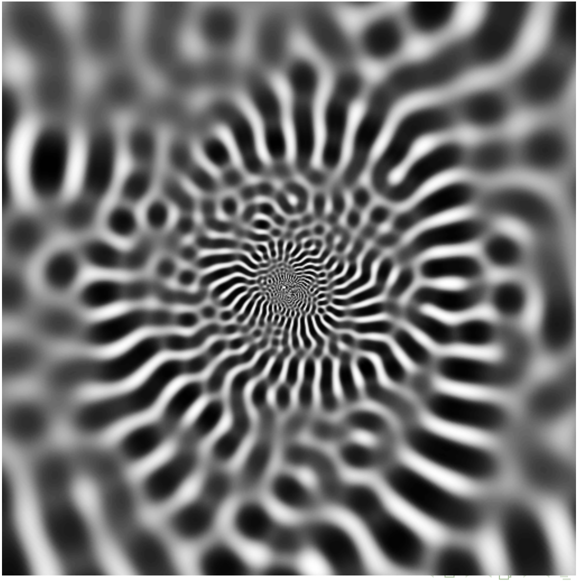

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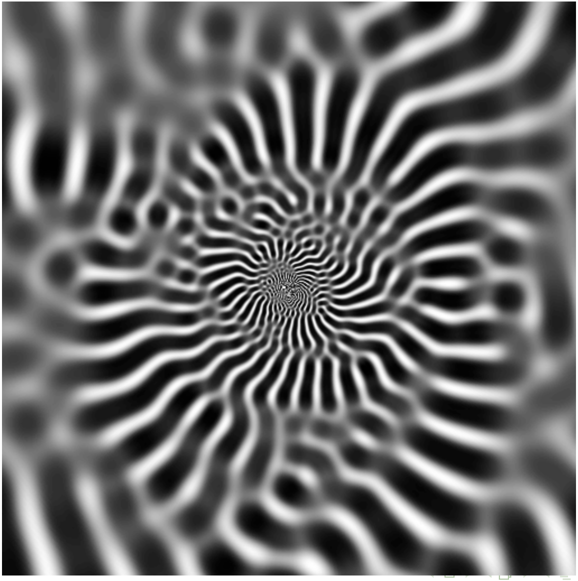

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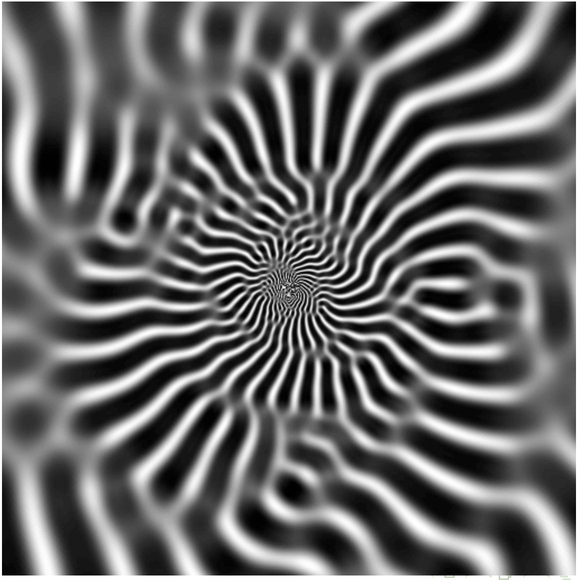
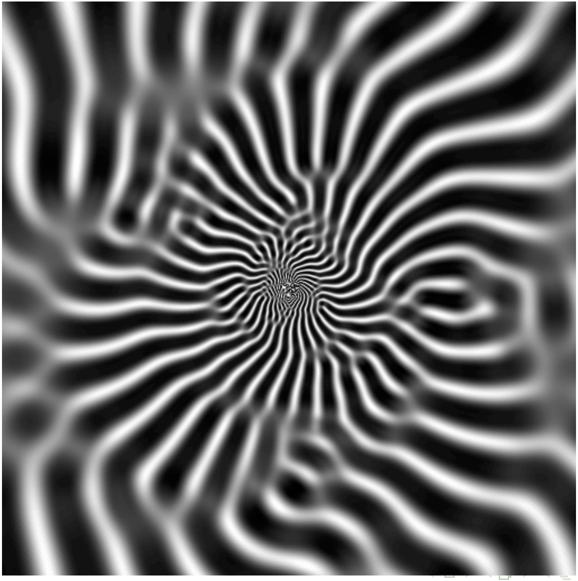
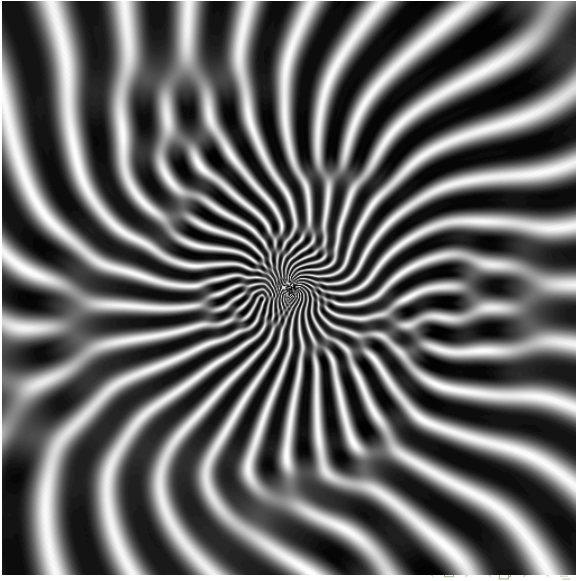


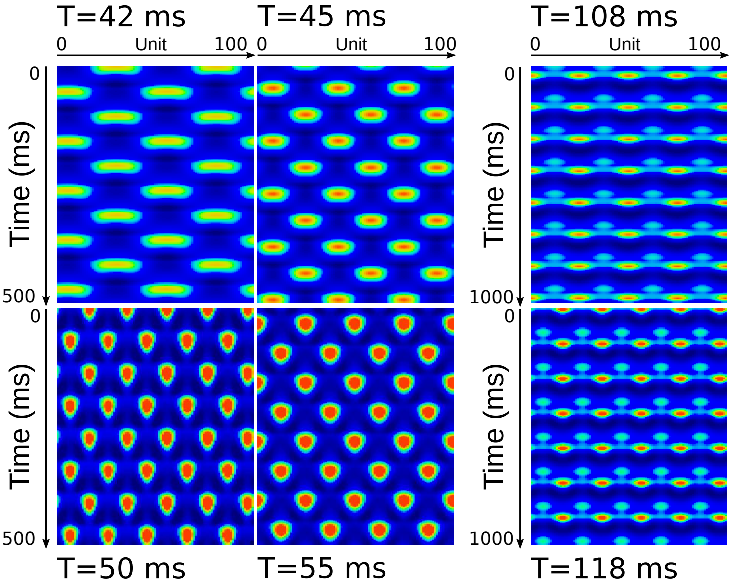
Evolution of 2D patterns (subjective coordinates)



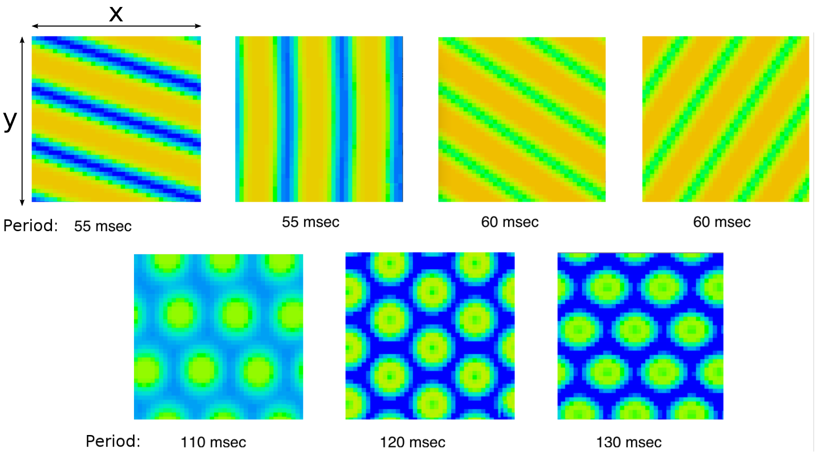
Evolution of 2D patterns (subjective coordinates)



1D patterns in time

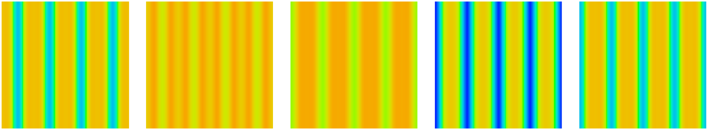


2D patterns: stripes and hexagons



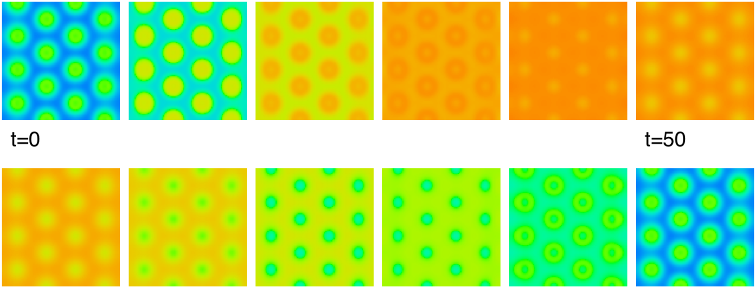
2D patterns: synchronous and period doubling

18 Hz : Symmetric, alternating stripes



t=0 t=12.5 t=27.5 t=40 t=55

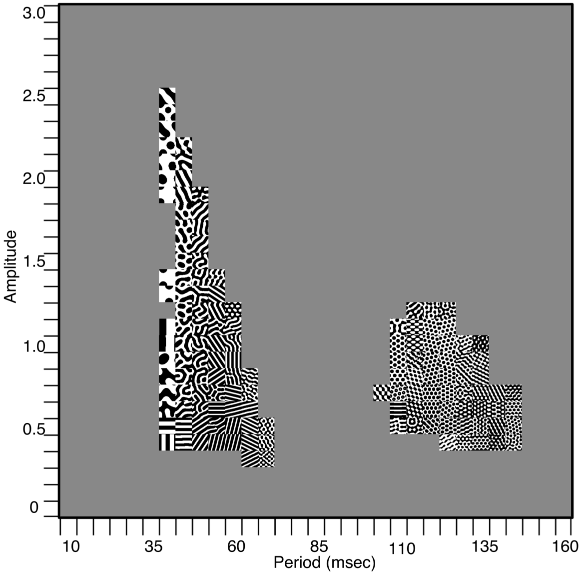
9 Hz : Hexagons



t=0 t=50

t=60 t=110

Frequency-dependent pattern formation



Stability

Stability analysis

- **Nonlinear**
  - Linearize at **spatially homogeneous** solution and examine stability
- Coefficients vary in time
  - When stimulated, there are no fixed points, perhaps fixed **orbits**?
  - Exploit periodicity and use Floquet theory to understand evolution
    - Numerically compute monodromy matrix, examine eigenvalues

Solving spatially homogeneous case

$$\dot{U}(x) = -DU(x) + F(KU(x) + GS(t))$$

If the system is spatially homogeneous, lateral interactions can be replaced with constants. This is a simpler 2D nonlinear system. Call this solution V.

$$\dot{V} = -DV + F(KV + S(t))$$

Linearizing around homogeneous solution

Once you have the spatially homogeneous solution V,

$$\dot{V} = -DV + F(KV + S(t))$$

decompose U into V, and a perturbation around V, which we will call Z.

$$\dot{Z} = -DZ + F'(K_o V(t) + S(t))(KZ)$$

Let $B(t) = -D + F'(K_o V(t) + S(t)) * K$, such that $\dot{Z} = B(t)Z$

Assessing stability of periodic orbits

The spatially homogeneous solution V is periodic

$$V(t) = V(t+T)$$

At critical points(orbits), Z is an ϵ departure from the spatially homogeneous solution.

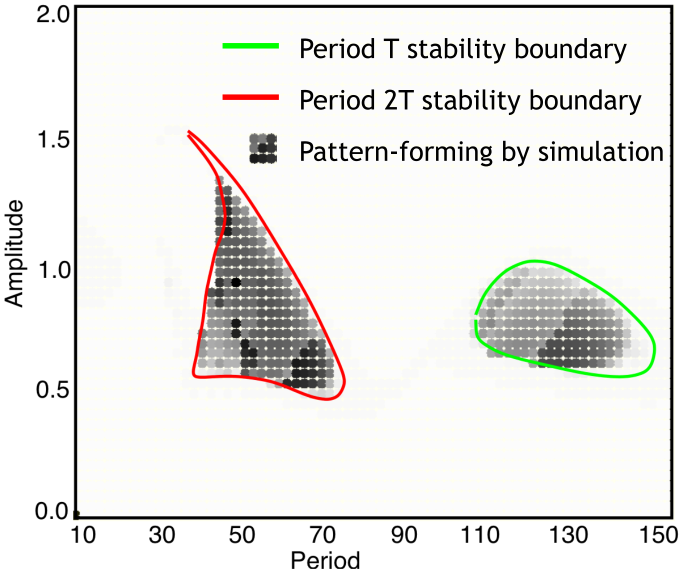
- Treat spatial eigenfunctions independently
- Examine how each eigenfunction evolves over one period

For a particular eigenfunction β, eigenvalues of the monodromy matrix will tell us whether β is growing.

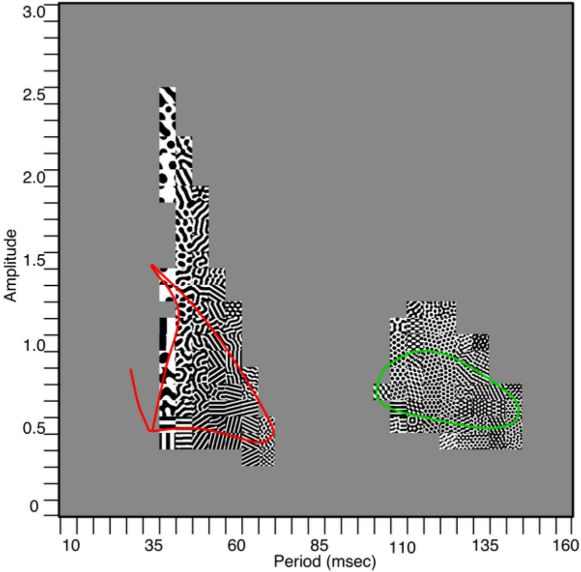
- Since the interactions are a convolution, the eigenfunctions are Fourier space.

$$Z_\beta(t+T) = M_\beta Z_\beta(t)$$

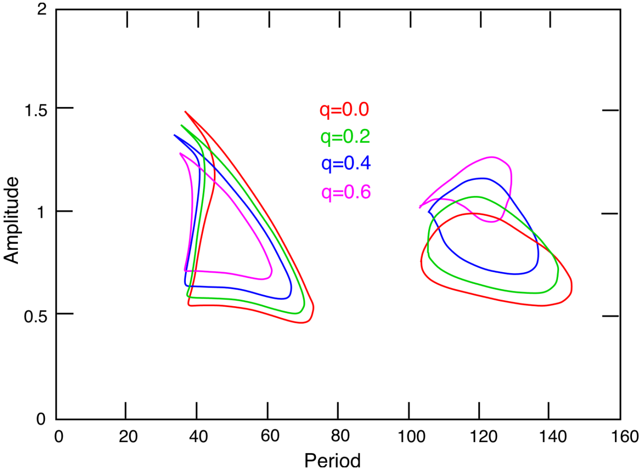
Stability analysis agrees with 1D simulation



2D simulation?



Parameter exploration: Feedforward Inhibition



$$q = \frac{g_i}{g_e}\text{: ratio of feed-forward inhibition and excitation}$$

Open problems addressed in this talk

- Simple models of visual hallucination **can** simulate flicker-phosphenes.

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- Simple models of visual hallucination **can** simulate flicker-phosphenes.
- Spatially uniform periodic stimuli may cause pattern formation by forcing the neural field into a pattern-forming periodic orbit.
- Resonant visual stimuli more readily induce patterns, but period-doubling pattern forming regimes are also favored.

Open questions

Modeling

- Better approximations of V1 network
  - Orientation: Bressloff et al?
  - Color: red-green effect in flicker, epilepsy
- Migraine?
- Epilepsy?

Experimental

- Psychophysics?
- Electrophysiology?

Acknowledgments

Coauthors

- **G. Bard Ermentrout**
- Matthew Stoffregen

Program in Neural Computation at CMU-Pitt CNBC

Supported by

- NIH funded Program in Neural Computation summer REU program
- NSF EMSW21-RTG0739261
- NSF DMS0817131

(this is the end of the talk)

Appendix

Retinotopic mapping of primary visual cortex

Retinotopic mapping of primary visual cortex

Hypothesis

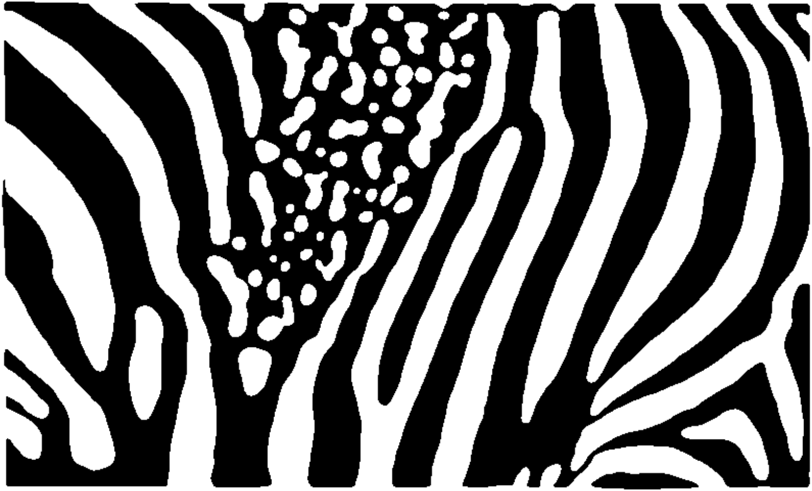
- Plane waves in V1 account for subjective patterns
- Periodic forcing with a uniform stimulus creates standing waves
    - Like Faraday waves?
- Can a neural field behave similarly?

Deegan, Merkt, Swinney,
Faraday waves in periodically forced fluid, Center for Nonlinear Dynamics, UT Austin.

Stability in a simplified model

Stripes or spots?

Stripes or spots?



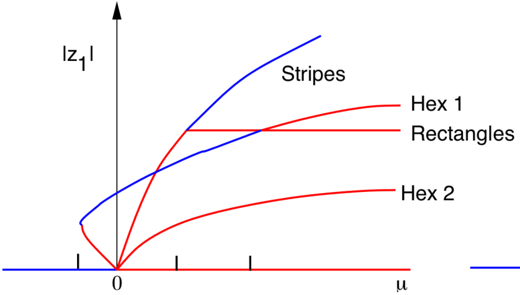
Stripes or spots?

Image credit: Farid Radjouh

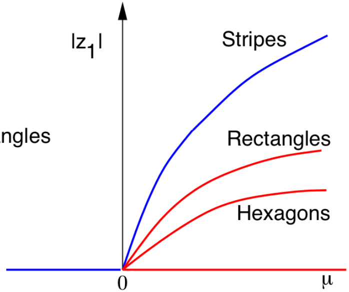
Sums of plane waves

Stripes or spots?

Low Frequency (7-10Hz)



High Frequency (15-30 Hz)



Bifurcation parameter μ is a function of model parameters $g_e, g_i, a_{ee}, a_{ie}, a_{ei}, \theta_e, \theta_i, \frac{T}{\tau_e}, \frac{\tau_i}{\tau_e}, \frac{\sigma_i}{\sigma_e}$. There is no closed form solution for μ. μ can also be expressed as a function of the eigenvalues of the monodromy matrix. When $\mu < 0$ the homogeneous solution is stable. As μ departs from 0 in the positive direction, we move in to pattern-forming regimes. Blue curves indicate stable patterns.

Big picture

- Spatially coupled, nonlinear systems exhibit complex resonance phenomena
  - Resonance associated with instability in e-i dynamics
- Nonlinearity and spatial coupling create multiple resonance peaks
  - Spatial patterns depend on frequency

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